Multistage Optimal PMU Placement in the Spanish Power Grid

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***Abstract*— This paper introduces a comprehensive framework aimed at optimizing the deployment of Phasor Measurement Units (PMUs) within real power grids. The methodology involves a sequential resolution of multiple optimization problems, specifically utilizing integer linear programming, to gradually achieve full observability of the power grid with the minimum necessary number of PMUs. The sequential optimization approach ensures that each step builds on the previous one, refining the placement of PMUs to enhance grid observability systematically. To validate the effectiveness of the proposed framework, the study focuses on the Spanish power transmission grid, which encompasses approximately 1300 buses. This underscores its applicability in real-world applications. The findings highlight the framework's capability to enable the systematic deployment of PMUs towards the full grid observability, providing insights into the costs and reliability of the monitoring infrastructure. This approach not only ensures optimal PMU placement but also facilitates better decision-making for grid planning, making it a valuable tool for power system operators.**

***Index Terms*—Phasor measurement unit (PMU), power system monitoring, smart grid, synchrophasor, wide area monitoring**

I. Introduction

The rapid growth of distributed energy resources (DERs) is bringing unprecedented changes in the dynamics and stability of power systems. Their proliferation introduces faster transients and induces phenomena that are challenging to identify with traditional grid monitoring systems like the Supervisory Control and Data Acquisition (SCADA) [1]. For this reason, to ensure grid reliability and resilience during the energy transition, advanced measurement devices are essential for the proper monitoring, protection, and control of modern power systems. This demand is further amplified by the growing role of artificial intelligence, requiring large amounts of data, and the increased automation and control of the power grid, requiring reliable sensors.

In response to these needs, phasor measurement units (PMUs) are in the process of being globally implemented by transmission system operators (TSOs) [2]. For this purpose, the optimal deployment of PMUs to achieve the full observability of power grids is a crucial challenge to address.

This objective can be reached through the installation of PMUs at every bus of the grid. However, this approach is impractical both economically and technically. Full observability can be obtained employing a much lower number of PMUs, strategically located [3].

This problem, known as the optimal PMU placement (OPP), was firstly addressed in [4] and then widely explored in the scientific literature. The latest contributions to this problem include considerations regarding small-signal [5], transient stability aspects of power grids [6], and the minimization of data communication bandwidth consumption through data pruning, in order to avoid communication congestion [7]. The problem of communication infrastructure was also explored in [8], proposing a multi-objective approach that accounts for resilience. The position of the PMUs may also influence the accuracy of measurements and estimations. For this reason, [9] developed an algebraic approach to improve state estimation from PMU data. The referenced works cover different theoretical aspects of the OPP problem. Implementing the solution of the OPP problem in real power grids requires splitting the installation into different stages. For this purpose, multistage OPP was initially proposed by [10], as a variation of the original OPP problem. The authors in [11] rejected the assumption of fixed PMU locations over the multistage installation process, and added to the model the possibility of relocating them. However, as pointed out in [12], the relocation may have a significant impact due to substation upgrades and communication infrastructure costs, leading to sub-optimal solutions.

Most multistage OPP models have been validated on benchmark power grids, with less than two hundred nodes, with the exception of [13], validating a fuzzy decision based multistage OPP in the Polish transmission grid.

Additional practical aspects evaluated in the literature include the exploitation of PMUs installed in boundary buses between different national grids [14], to reduce the overall costs. A multi-objective formulation considering fault-location observability is applied to the Oman grid in [15]. However, a systematic approach for the TSOs to implement the multistage OPP to large real power grids is lacking. This paper aims at filling this gap, by presenting a comprehensive framework for the multistage OPP, tailored for real power grids.

The proposed framework, presented in the next sections, consists in the sequential solution of different optimization problems, returning the optimal PMU installation strategy.

This approach aims to support TSOs not only in the initial planning phase, but also during intermediate stages of the PMU installation process, ultimately achieving full observability in the power grid. The framework is applied and validated using data from the Spanish power transmission grid.

The remainder of the paper is organized as follows: Section II provides the theoretical background on the OPP problem. Section III describes the proposed multi-stage placement framework. Section IV presents the results of applying this framework to the Spanish power grid. Finally, Section V draws the main conclusions.

II. Theoretical Background

The optimal PMU placement problem involves determining the optimal number and locations of PMUs to be installed in a power grid to ensure its observability. A power system is considered observable when the state variables, i.e., the voltage phasor magnitude and phase angle, are known at all buses.

If a PMU is installed on bus i, then bus i and all adjacent buses become observable. This observability rule directly follows from Ohm's law, provided that the physical characteristics of the grid infrastructure are well known [16]. For example, a PMU installed on a generic bus i, connected to bus j, can measure the voltage phasor V_i and the current phasor I_{ij}. Knowing the transmission line impedance Z_{ij}, the voltage phasor of the adjacent bus V_j can be calculated using Ohm's law.

Additional observability rules can be introduced in the presence of zero injection buses (ZIB), interconnection buses where no current is injected. The presence of these buses, leveraging Kirchhoff's current law, reduces the minimum number of PMUs required for full observability.

The formulation of the OPP problem involves modelling the power grid as an undirected graph $G = (\mathcal{B}, \mathcal{E})$, where $\mathcal{B}$ is the set of buses and $\mathcal{E}$ the set of edges corresponding to transmission lines connecting the buses [17]. The PMU placement $\mathcal{P}$ is a subset of nodes in $\mathcal{B}$, representing the location of the installed PMUs. The system observability $\mathcal{O} = f(\mathcal{P})$ is another subset of $\mathcal{B}$, dependent on $\mathcal{P}$. The system is considered fully observable when $\mathcal{O} = \mathcal{B}$. In graph theory, this is equivalent to state that the placement $\mathcal{P}$ is a cover for G.

Given the graph G and two placements $\mathcal{P}$ and $\mathcal{P}'$, with their cardinalities $|\mathcal{P}|$, $|\mathcal{P}'|$, if

$$|\mathcal{P}| \leq |\mathcal{P}'|, \quad \forall\, \mathcal{P}' \subseteq \mathcal{B} \tag{1}$$

then the placement $\mathcal{P}$ is defined as a minimum cover, representing the minimum number of PMUs required for topological observability. Finding a minimum cover requires the definition of the following observability rules, emerging from the Ohm and Kirchoff's laws. To define these rules, we introduce the concept of monitored node, which is a node equipped with a PMU. These rules can be formally stated as:

- If node i is monitored, then all adjacent nodes $\mathcal{A}_i$ are observable. In other words, if $i \in \mathcal{O}$, then the subgraph $\Gamma(i)$ consisting of its adjacent nodes, defined as
$$\Gamma(i) = (\mathcal{A}_i, \{(i,j) | (i,j) \in \mathcal{E} \text{ and } i,j \in \mathcal{A}_i\}) \tag{2}$$
is also observable, i.e. $\mathcal{A}_i \in \mathcal{O}$.
- If node i is monitored and all nodes in $\Gamma(i)$, excluding one, are observable, then all nodes in the subgraph are observable, i.e. $i \cup \mathcal{A}_i \in \mathcal{O}$.

A graph may have a large number of potential minimum covers, increasing with the number of nodes. Finding all the minimum covers of a graph is a NP-complete problem and is challenging to solve for real transmission power grids with hundreds of nodes. However, being it a minimization problem, specific solutions can be extracted from the entire set of solutions through integer linear programming (ILP).

Considering that the PMUs cannot be installed in real power systems all at once, the installation should be planned in different phases. For this reason, multistage approaches are required for the practical PMU deployment in real power grids.

Next section describes the proposed framework for the multistage optimal PMU placement in large, real power grids.

III. Methodology

The framework described in this section can be applied to plan the gradual deployment of PMUs in real transmission power grids, until achieving the full observability of the system with the minimum number of PMUs required. The procedure for the proposed multistage optimal PMU placement is summarized in the Pseudocode 1.

Pseudocode 1 Multistage OPP

```
Input: Grid topology
Output: Multistage optimal PMU placement
  Solve the minimum PMU placement problem (N_PMU)
  Set the constraint Σ_{i∈N_bus} b_i = N_PMU
  if Objective function (OF) is cost minimization then
      Solve the cost minimization problem
      Store the subset of PMU locations S_cost
  else if OF is redundancy maximization then
      Solve the redundancy maximization problem
      Store the subset of PMU locations S_red
  end if
  Set the constraint i ∈ S_cost or S_red
  while p ≤ N_PMU do
      Solve optimal partial observability problem for stage p
      Store the PMU locations in stage p
  end while
  Output multistage optimal PMU placement solutions.
```

The requisite input data includes the topology of the power grid under analysis, and, if available, the positions of ZIBs and existing SCADA systems. The output returns the location of the PMUs to be installed at each stage of the process. The procedure involves different formulations of the OPP problem, described below [6].

A. Minimum number of PMUs

The first step involves solving the basic OPP problem outlined in (3). This aims to determine the minimum number of PMUs, denoted as N_{PMU}, essential for achieving full observability of the power grid.

$$\min_{\boldsymbol{b}} \quad \sum_{i \in N_{\text{bus}}} b_i \tag{3a}$$

$$\text{s.t.} \quad \boldsymbol{b} + \boldsymbol{A} \cdot \boldsymbol{b} \geq \mathbf{1} \tag{3b}$$

$$b_i \in \{0, 1\} \quad \forall\, i \tag{3c}$$

where $\boldsymbol{b}$ is the vector of binary variables representing the position of the PMU, $\boldsymbol{A}$ is the adjacency matrix and N_{bus} is the total number of buses in the power grid. The i-th element of $\boldsymbol{b}$ equals 1 if a PMU is installed at node i and 0 otherwise. Constraint (3b) enforces full observability, indicating that each grid bus must either have a PMU or be adjacent to a node with a PMU, following the previously defined observability rules. The output of this phase is N_{PMU}, representing the minimum number of PMUs necessary for complete system observability.

Considering that the number and locations of ZIBs are often limited and not available in power grid datasets, the ZIB constraint is omitted. This may slightly affect the results, as the minimum number of PMUs required in a system when considering ZIBs is lower. This problem, lacking additional constraints or a more specific objective function, may yield multiple solutions, especially for large systems, as the same minimum number of PMUs can correspond to different minimum covers of the graph representing the grid, also known as power dominant sets [18]. Consequently, the only useful output is N_{PMU}, whereas the set of optimal locations obtained is equally valid as the alternative ones, until an additional objective function is introduced.

For this reason, an additional optimization problem is solved choosing a metric for assessing the quality of the solution, that is incorporated as the new objective function. Hence, following the determination of N_{PMU}, the subsequent step involves imposing the constraint $\sum_{i \in N_{\text{bus}}} b_i \geq N_{\mathrm{PMU}}$ and changing the objective function to the chosen one. In this paper, the objective functions chosen are two: cost minimization and redundancy maximization.

B. Cost minimization

Within all the power dominant sets, there is one with the minimum cost. In order to find this solution, the cost of PMUs should be characterized as a function of their location and characteristics. The location of the PMU installation may affect the price in different ways, described in the literature [19], including the instrument transformer, eventual shutdown costs or communication infrastructure. Hence, each PMU location should, in principle, be characterized by a specific price $\lambda_{\mathrm{PMU},i}$. However, as this information is not openly available, the literature suggests to use an estimation of the costs that depends on the number of measurement channels required by each PMU to cover, i.e. for all the branch currents connected to the installation bus. Given a constant price λ_{PMU} for a PMU device, each branch measurement channel is supposed to add 10% to the original cost.

Hence, with this assumption, the problem can be formulated as

$$\min_{\boldsymbol{b}_{\text{cost}}} \quad \sum_{i \in N} \lambda_{\mathrm{PMU}} (1 + 0.1 \sum_{i} a_{ij}) b_{\text{cost},i} \tag{4a}$$

$$\text{s.t.} \quad \boldsymbol{b}_{\text{cost}} + \boldsymbol{A} \cdot \boldsymbol{b}_{\text{cost}} \geq 1 \tag{4b}$$

$$\sum_{i \in N_{\text{bus}}} b_{\text{cost},i} \geq N_{\mathrm{PMU}} \tag{4c}$$

$$b_{\text{cost},i} \in \{0, 1\} \tag{4d}$$

where a_{ij} is the element of the adjacency matrix $\boldsymbol{A}$ connecting bus i and j, that is 1 if the two buses are connected and 0 otherwise.

In this case, the vector of decision variables $\boldsymbol{b}_{\text{cost}}$ is renamed to highlight that this solution is different than the original one $\boldsymbol{b}$, obtained in the first step.

C. Redundancy maximization

Another potential choice for the objective function is the maximization of observability redundancy. This choice improves the resilience of the monitoring system to outages, while maintaining the minimum number of PMUs to guarantee the full observability. For this purpose, the System Observability Redundancy Index (SORI) is employed [10]. This index counts the number of times each bus of the power grid is observable, considering its adjacency with buses equipped with PMUs.

$$\max_{\boldsymbol{b}_{\text{red}}} \quad \sum_{i \in N_{\text{bus}}} \boldsymbol{A}_i \cdot \boldsymbol{b}_{\text{red}} \tag{5a}$$

$$\text{s.t.} \quad \boldsymbol{b}_{\text{red}} + \boldsymbol{A} \cdot \boldsymbol{b}_{\text{red}} \geq 1 \tag{5b}$$

$$\sum_{i \in N_{\text{bus}}} b_{\text{red},i} = N_{\mathrm{PMU}} \tag{5c}$$

$$b_{\text{red},i} \in \{0, 1\} \tag{5d}$$

Where $\boldsymbol{A}_i$ is the row of the adjacency matrix corresponding to the i-th bus.

The sets of PMU locations corresponding to the optimal $\boldsymbol{b}_{\text{cost}}$ and $\boldsymbol{b}_{\text{red}}$ are denoted as $\mathcal{S}_{\text{cost}}$ and $\mathcal{S}_{\text{red}}$.

D. Multistage optimization

Once the optimal location is found, either by choosing the minimum cost or the maximum redundancy strategy, the multistage optimization process can start. Considering the large cost of installing all the PMUs required to achieve full observability, this is unfeasible for the optimal PMU placement in real power grids. For this reason, it should be carried out through multistage installations [20].

The multistage installation requires the definition of a new vector of variables, $\boldsymbol{\alpha}$. The generic element α_i is equal to 1 if bus i is observable, and 0 otherwise. Furthermore, this problem requires the optimal location of the PMUs from the solution of either the minimum cost (4) or the maximum redundancy (5) problem, denoted as $\boldsymbol{y}_{\text{opt}}$.

The multistage optimization consists in solving the problem (6a) for each stage p, starting with $p = 1$. In other words, the number of PMU locations found at each stage is equal to p.

The generic optimization problem for the stage p is

$$\max_{\boldsymbol{b}_{\text{part}}, \boldsymbol{\alpha}} \quad \sum_{i \in N_{\text{bus}}} \alpha_i \tag{6a}$$

$$\text{s.t.} \quad \boldsymbol{b}_{\text{part}} + \boldsymbol{A} \cdot \boldsymbol{b}_{\text{part}} \geq \boldsymbol{\alpha} \tag{6b}$$

$$\sum_{i \in N_{\text{bus}}} b_{\text{part},i} = p \tag{6c}$$

$$\overline{\boldsymbol{y}}_{\text{opt}}^T \cdot \boldsymbol{b}_{\text{part}} = 0 \tag{6d}$$

$$b_{\text{part},i} = 1 \quad i \in \mathcal{S}_{\text{part},p-1}, \ p > 1 \tag{6e}$$

$$b_{\text{part},i}, \alpha_i \in \{0, 1\} \tag{6f}$$

The constraint (6e) is required to keep track of the PMU locations found in the previous step $p - 1$, belonging to the subset $\mathcal{S}_{\text{part},p-1}$, and fix their position as a new PMU is added to the grid.

On the other hand, the constraint (6d) limits the set of partial solutions to the subset of optimal locations $\mathcal{S}_{\text{cost}}$ or $\mathcal{S}_{\text{red}}$ found by solving either the minimum cost or the maximum redundancy problem. $\overline{\boldsymbol{y}}_{\text{opt}}$ represents the bitwise negation of the original binary vector representing the optimal PMU locations. Its product with $\boldsymbol{b}_{\text{part}}$ ensures that the PMU locations found at each stage p are within the subsets $\mathcal{S}_{\text{cost}}$ or $\mathcal{S}_{\text{red}}$. In set notation $\mathcal{S}_{\text{part},p} \in \mathcal{S}_{\text{cost}} \vee \mathcal{S}_{\text{red}}$.

E. Single PMU failure

Potential failures of the PMUs pose a risk to the complete observability of the power grid, leading to significant challenges when the power system control and automation rely heavily on PMU measurements [21]. To enhance the resilience of the monitoring system to single PMU outages, adjustments can be made to the problem formulation. Specifically, the constraint (3b) can be modified to ensure that each bus in the system is observable at least twice, even in the event of a single PMU outage. The minimum number of PMUs required in this case would be N_{rel}. This information proves valuable for assessing whether the reliability improvement justifies the associated costs. Similar to problem (3), the multitude of potential PMU combinations satisfying the constraints presents a challenge, and the choice will ultimately depend on the objective function, representing the parameter that defines the superiority of one solution over another.

IV. Case Study

The proposed multistage optimal PMU placement framework is examined using real Spanish transmission grid data, from the SciGridKit project [22]. The system has $N_{\text{bus}} = 1275$ buses and the initial price of a PMU λ_{PMU} is assumed to be €$20,000$, covering device and installation expenses.

In the first step, the solution of the problem (3) returns the minimum number of PMUs necessary for achieving full observability. For the Spanish grid, the solution obtained is $N_{\text{PMU}} = 417$, corresponding to the 32.7% of the total number of grid buses. This number can be potentially reduced with additional information from the TSO regarding ZIB locations, allowing the introduction of ZIB constraints. The reliability problem is subsequently addressed to find the number of additional PMUs required to ensure the resilience of the grid observability to single PMU outages. The solution results in $N_{\text{rel}} = 924$ PMUs, more than the double of the minimum.

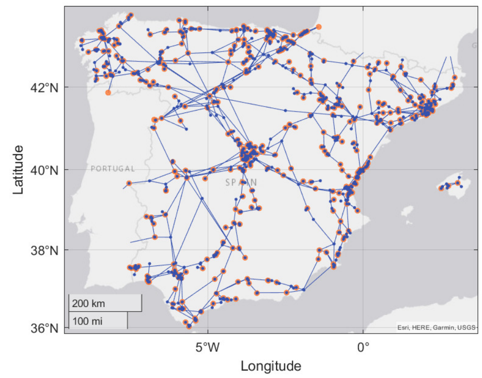

Fig. 1. Optimal PMU placement in the Spanish power transmission grid. Highlighted in orange are the locations where PMUs are strategically installed.

TABLE I
OPP INDICES FOR THE SPANISH GRID

N_{bus}	N_{PMU}	N_{rel}	SORI_{min}	SORI_{max}	Cost_{min} [M€]
1275	417	924	1443	1811	10.39

The second step involves identifying the optimal locations guaranteeing the minimum cost (4) or the maximum observability redundancy (5), by constraining the number of PMUs to the minimum calculated in the previous step.

The optimal locations resulting from the minimum cost solution are depicted in Fig. 1.

Table I resumes the key indices computed in these initial two steps. The minimum cost solution requires 10.39 M€, while the maximum redundancy solution leads to a SORI of 1811. The trade-off between the two solutions obtained from (4) and (5) is computed in terms of SORI and costs:

$$\Delta\text{SORI} = \text{SORI}_{(5)} - \text{SORI}_{(4)} = 368 \tag{7}$$

$$\Delta\text{Cost} = \text{Cost}_{(4)} - \text{Cost}_{(5)} = 0.74 \text{ M€} \tag{8}$$

The SORI_{min} index in Table I represents the worst case location of the PMUs in terms of observability redundancy. For the Spanish grid, the minimum SORI is $\text{SORI}_{\text{min}} = 1443$. It is obtained by minimizing the objective function of (5), instead of maximizing it. This index proves valuable in assessing the goodness of a solution in terms of reliability, providing a base case for comparison. The SORI provides an idea of the resilience of the system to single PMU outages. Indeed, the maximum redundancy solution leads to $\text{SORI}_{\text{max}} = 1811$. This translates in a total of 400 buses that are observable at least twice and therefore can withstand the loss of a single

PMU, without affecting the grid observability. This number falls from 400 to 141 when the minimum cost strategy solution is considered. Combining the results reveals that the cost of assuring the redundancy on 259 buses is around 0.74 M€. Once the optimal locations have been identified, the third step consists in carrying out the multistage optimization, starting from the first PMU, $p = 1$. The evolution of the system observability is shown through the partial observability plot, where the partial observability is computed as the fraction of the observable buses at stage p over the number of PMUs in the system. Fig. 2 shows the evolution of the system partial observability in the Spanish grid for both the minimum cost and maximum redundancy strategies.

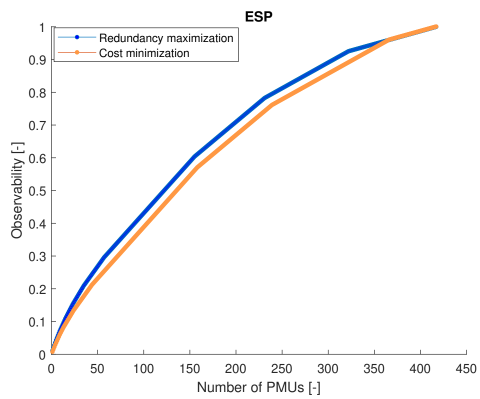

Fig. 2. Partial observability plot illustrating the outcomes of different PMU placement strategies.

By observing the evolution of the partial observability index for the two strategies, Fig. 2, it is shown that their difference at intermediate stages is almost constant, approximately 4% of the total. This corresponds to a difference in the observability of 51 buses. This result is strictly dependent on the cost function choice. In this case, the constant difference in intermediate stages is caused by the dependence of the current cost function on the topological characteristics of the network, i.e. the number of adjacent buses. However, if prices change over different buses, meaning that there is a $\lambda_{\mathrm{PMU},i}$ for each bus, the difference between the cost minimization and maximum redundancy strategy may change over the stages.

The plot of the evolution of the partial observability provides valuable insights for TSOs in the installation planning phase. This is explained with the example shown in Fig. 3, that is a zoom of the partial observability plot for the values of interested. In this case, it is assumed that the TSO is interested in reaching the 70% of grid observability in the next stage. From the partial observability plot, by tracing an horizontal line corresponding to the chosen observability level, the TSO can evaluate the number of PMUs required to pursue the chosen strategy. For example, in this case study, given an observability level goal of the 70%, 196 PMUs are needed when following the redundancy maximization strategy and 219 for the minimum cost.

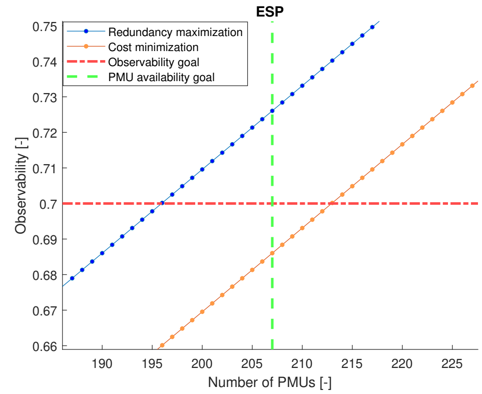

Fig. 3. Utilization of the partial observability plot for planning installation stages. The vertical line indicates a fixed amount of PMUs available, while the horizontal line indicates the desired observability level.

This result may appear counter-intuitive, but the plot showing the cost versus observability, in Fig. 4, demonstrates that the minimum cost strategy guarantees the lowest costs only once the full observability is reached, but not necessarily during the intermediate stages. It should be also noted that although the curves appear close, their cost difference is economically significant, around 0.5 M€.

This aspect should be taken into duly account, especially for long-time planning horizons and high interest rates, since it may lead to unexpectedly larger costs at the end of the installation process.

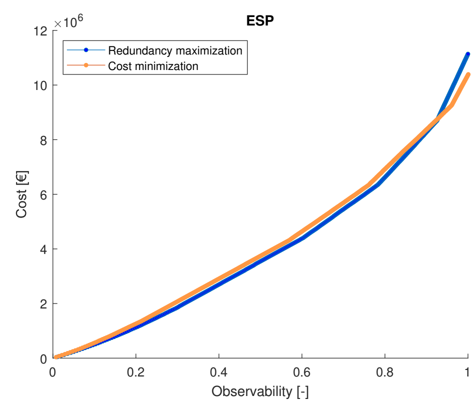

Fig. 4. Costs versus partial observability plot for different PMU placement strategies.

Similarly, a TSO may have a certain availability of PMUs and may have to assess the degree of observability achievable them. The partial observability level corresponding to the PMU availability is obtained by drawing a vertical line corresponding to that availability. For example, assuming 217 PMUs are available, the corresponding observability levels are 68% for the minimum cost strategy and 72% for the maximum redundancy one. Finally, an evaluation of the resilience level of the system at intermediate stages can be carried out by evaluating the number of buses with redundant observability

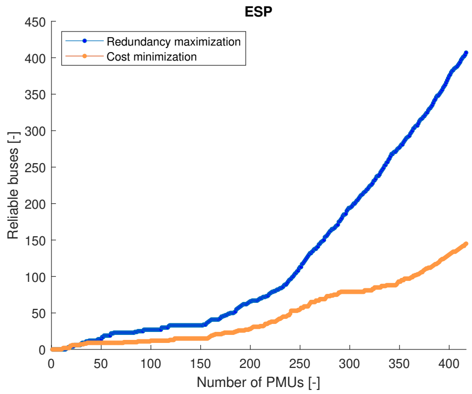

Fig. 5. Evolution of the number of reliable buses for different OPP strategies.

sources in each stage. This is depicted in Fig. 5 for both selected strategies. It is evident from the data that the maximum redundancy strategy ensures a superior level of fault tolerance and resilience across all stages.

V. Conclusion

The paper introduces a practical PMU deployment framework tailored for real power grids, offering valuable guidance to TSOs during the planning phase. The framework suggests an optimal strategy, considering different objective functions, for the gradual installation of PMUs in transmission grids, with a primary goal of achieving full observability. Validation of the framework was conducted on the Spanish transmission grid, providing concrete insights into its applicability and effectiveness. The study determined the minimum number of PMUs required for full observability in a power grid, finding that about one-third of the nodes need PMUs. The order of installation in the optimal locations found is determined through the proposed multi-stage optimization. The framework's practical value was shown by illustrating how a TSO can use it to determine the number of PMUs needed for a desired observability level, such as 217 PMUs for 68% observability under a minimum cost strategy. Additionally, applying the multistage approach to the N-1 reliability case revealed increasing differences in the number of reliable buses between maximum redundancy and minimum cost strategies as installation progresses. As a direction for future research, a potential option is extending the application of the proposed framework to various European transmission grids, enabling a comprehensive comparison.

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